A Combinatorial Approach to Shape Optimization of High Field Magnets

Andrea G. Chiariello, Alessandro Formisano, Francesco Ledda, Raffaele Martone, Francesco Pizzo

Seconda Università degli Studi di Napoli, Department of Industrial and Information Technology, Aversa, I-81031 Italy
francesco.pizzo@unina2.it

The optimization of the magnets' shape can be very demanding because of the number of field map evaluations required especially when a high accuracy is needed. Here, in the limits of linearity assumption, a new approach is proposed based on the projection of the coil geometry in a finite dimension functional basis. Then the associated magnetic contributes can be computed in advance and any possible solution can be examined just by combining a suitable set of elementary contributes. The advantage in computing burden and accuracy is assessed in the case of challenging magnets for Tokamak devices.

***Index Terms*— Optimization, Magnetostatics, Fusion Reactor Design.**

I. INTRODUCTION

CONTROLLED THERMONUCLEAR FUSION (CTF) is one of the most promising ways to produce energy from fusion reactions on the earth. In CTF, atoms' nuclei fuse together releasing high amount of energy; the process is the reverse of nuclear fission, nowadays used in common nuclear plants. One of the main problems in the development of a fusion machine is the confinement of the plasma generated to obtain the fusion reactions. The most promising fusion devices are the so called Tokamaks [1], which confine plasma in a toroidal camera by means of high magnetic fields.

The typical tokamak magnetic system includes a Toroidal Field Coil System (TFC) and a Poloidal Field Coil System. The toroidal field mission is to confine and stabilize the plasma. Due to practical reasons the TFC is designed as a finite number of magnets distributed in the toroidal directions; as a consequence the magnetic field cannot be uniform and a ripple component arises. The ripple is the main origin of a number of plasma instabilities [2], therefore, it should be suitably limited. In order to reduce the ripple, several measures can be taken, falling in two classes: passive (ferromagnetic inserts [3]) or active measures (as the Active Compensating Coils, ACC [4]).

The ACC design is a challenging task, falling in the class of electromagnetic shape optimization problems that typically are quite demanding because require a high number of objective function evaluations.

In this paper a new methodology able to face with the ACC shape design problem is proposed, under the assumption of absence of ferromagnetic material. The approach is able to take advantage of a suitable decomposition of the ACC geometry in a finite dimension function basis. In this way, the magnetic field due to each basis element can be computed in advance just once. Therefore, any possible ACC shape is just a suitable combination of basis elements serially connected. In the following it will be discussed how the procedure is able to reduce the computational burden from an exponential to a linear cost. Finally, the methodology is quite general because it can be adopted in a wide class of shape optimization problems.

II. MATHEMATICAL MODEL

In fig.1 a qualitative behavior of the B_φ distribution of a typical tokamak characterized by 18 TF magnets is shown with respect to the toroidal angle. An effective index able to quantify the ripple in a cylindrical coordinates system (R, φ, Z) is:

$$R_{ipple}(R,Z) = \left({B_\varphi}^{Max} - {B_\varphi}^{min}\right) / \left({B_\varphi}^{Max} + {B_\varphi}^{min}\right) \qquad (1)$$

where the Max and Min are evaluated along the circumference (R, Z) centered in r=0.

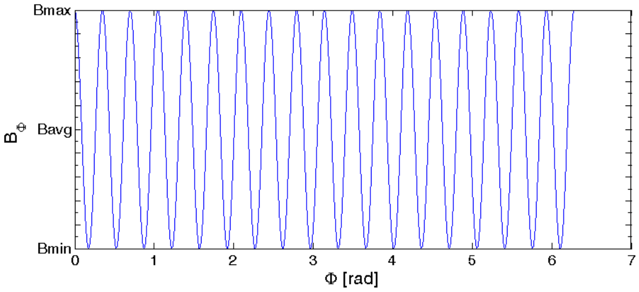

Fig. 1. Example of ripple phenomenon for a periodicity of 18 TFCs.

ACC could be manufactured in a quite similar way as TFC, by winding a number of series connected conductor turns and allocating them between the TFC and the vacuum vessel. A possible ACC design includes a number of coils placed between the TFC and the plasma chamber (fig. 2), closed outside and driven by opposite currents. The number, position and shape of the coils are to be optimally designed.

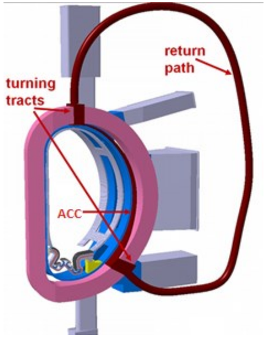

Fig. 2. Example of ACC closed to a TFC (artistic view).

In the recent past a procedure has been proposed [5] to face with such problem. The main advantage of the procedure was the possibility to calculate in advance the magnetic contributes because the shape of each possible conductor was fixed a priori.

In this paper a new strategy is proposed, able to face with the problem of shape optimization, taking advantages from the pre-calculation. Because of manufacturing constraints, each conductor is assumed to be composed by a finite number of elementary parts serially connected (because parts of the same cable) at some of the pre-assigned connection point (see fig. 3). In this way, the magnetic field of a coil with a current I, can be just evaluated by superposition of the contributes of the elements of the basis:

$$\mathbf{B}(\mathrm{R},\mathrm{Z}) = \mathrm{I}\sum_{\mathrm{k}=1}^{\mathrm{Ns}-1}\mathbf{G}_{\boldsymbol{k}}(\mathrm{R},\mathrm{Z}) \qquad (2)$$

where $\mathbf{G}_{\mathbf{k}}$ is the contribute of the k-th element (see fig. 3).

The link between adjacent elements is allowed by a finite number of N_s connection stations and, at the k-th station ($k=1,\dots,N_s$) by a finite number of $N_L(k)$ connection points $(R(j,k), Z(j,k))$, $j=1,\dots,N_L(k)$.

In order to guarantee the regularity of the coil shape, each element is designed to reach the stations in an orthogonal way (fig. 3). A polynomial cubic spline is well suited to meet such constraint because of its four degrees of freedom.

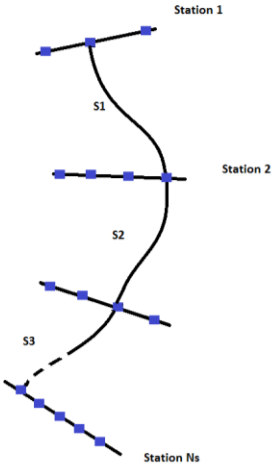

Fig. 3. Example of ACC conductor shape.

The magnetic contribute of each spline can be computed by carrying out a discretization of the spline into a number of small filamentary segments which analytical formula [6] is obtained by line integration of Biot-Savart law.

In general, the total number of possible conductor's shapes is $\mathrm{N}_{\mathrm{SH}} = \prod_{k=1}^{\mathrm{N_s}} \mathrm{N}_{\mathrm{L}}(\mathrm{k})$. Actually each shape is just a suitable connected subset of basis shape elements, which total number is limited to $\mathrm{N}_{\mathrm{T}} = \sum_{\mathrm{k}=1}^{\mathrm{N_s}-1} \mathrm{N}_{\mathrm{L}}(\mathrm{k})\mathrm{N}_{\mathrm{L}}(\mathrm{k}+1)$. Therefore, if an exhaustive optimal research is adopted, the new procedure is able to reduce the computational burden from the order of N_{SH} to the order of N_T (see next section for a numerical example). However, when the number of feasible solutions of the discrete research domain exceed the actual computing capability, a stochastic optimization algorithm can be used.

In the full paper will be shown how it is possible to formulate the optimization problem as a minimization of a scalar function of a single integer variable.

III. EXAMPLE OF APPLICATION

In order to assess the proposed algorithm, a simple example applied to FAST (Fusion Advanced Study Torus) tokamak [5] is here presented. In this digest, for lack of room, the shape of a single conductor fed by the same current of one TFC conductor will be optimized. In the full paper more complex configurations, with more conductors and much more degrees of freedom will be shown.

The geometry of the TFCs is assumed to be the same of [5] (fig. 2). The number of stations N_S is 9. The number of connection points is assumed to be equal for each station ($N_L = 3$). The total number of magnetic contributes (splines) to be analyzed with an exhaustive research ($\mathrm{N}_{\mathrm{SH}} = 157464$) strongly reduces ($\mathrm{N}_{\mathrm{T}} = 72.$) if adopting the new procedure. The objective function is the value of the ripple itself, evaluated at r=1.82 m, z=0 m.

After an exhaustive research of the optimum the ripple phenomenon is reduced from a nominal value of 1.97 10^{-4} to a minimum value of 1.75 10^{-4} (reduction of 11.46 %).

The resulting shape is the one shown in fig. 4.

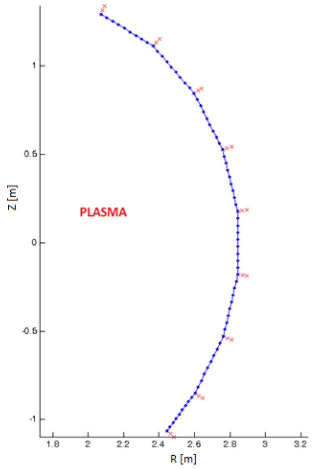

Fig. 4. Optimal solution for the example of application

IV. ACKNOWLEDGEMENTS

This work was partially supported by EU under the Contract of Association of EURATOM/ENEA/CREATE, and by the Italian MIUR under PRIN grant 2010SPS9B3.

V. REFERENCES

[1] Wesson, John. Tokamaks. Vol. 149. Oxford University Press, 2011.
[2] P. C. de Vries et al., Effect of toroidal field ripple on plasma rotation in JET. Nuclear Fusion, vol 48, n° 3, 2008.
[3] A. Portone et al., The ITER TF coil ripple: Evaluation of ripple attenuation using Fe insert and of ripple enhancement produced by TBM, Fusion Engineering and Design, vol. 83, 2008, pp. 1619-1624.
[4] G. Calabrò et al., Toroidal Field Ripple reduction studies for ITER and FAST, Fusion Engineering and Design, vol. 84, 2009, pp. 522-525.
[5] G. Calabrò' et al., "Optimal Design of the Correction Coils in Tokamak Fusion Devices", Proc. of OIPE Conference, Delft, 2014
[6] A. G. Chiariello, A. Formisano, R. Martone, Fast magnetic field computation in fusion technology using GPU technology, Fusion Engineering and Design, vol. 88, 2013, pp. 1635-1639.